

Comment on "Two Disks in a Box"
[R. J. Speedy, Physica A **210**, 341 (1994)].

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In contradiction to the widely-held view that phase transformations can only be associated with *infinite* systems, Speedy¹ showed that the partition function for a periodic *two*-disk system has a van der Waals' loop resembling, qualitatively, that found by Alder and Wainwright in their investigations of the melting-freezing transitions of hard disks and spheres some thirty years ago². This analog, as well as its extension to nonsingular repulsive potentials, has been noted before. In 1963, Alder, Hoover, and Wainwright³ called attention to the van der Waals behavior of two hard disks, using periodic boundary conditions suited to the close-packed triangular lattice. Compare the Figure in Reference 3 with Speedy's Figure 1. It was pointed out in Reference 3 that the pressure and density for the two-body transition lie within 10% of those estimated for hard disks in the thermodynamic limit⁴. Similar analytic results were pointed out for periodic systems of a few hard squares⁵.

References:

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- 1. R. J. Speedy, "Two Disks in a Box", *Physica A* **210**, 341 (1994).
- 2. See, for instance, the discussion in Chapter 6 in W. G. Hoover, **Computational Statistical Mechanics** (Elsevier, Amsterdam, 1991).
- 3. B. J. Alder, W. G. Hoover, and T. E. Wainwright, "Cooperative Motion of Hard Disks Leading to Melting", *Physical Review Letters* **11**, 241 (1963).
- 4. W. G. Hoover and F. H. Ree, "Melting Transition and Communal Entropy for Hard Spheres", *Journal of Chemical Physics* **49**, 3609 (1968).
- 5. W. G. Hoover and B. J. Alder, "Studies in Molecular Dynamics. IV. The Pressure, Collision Rate, and Their Number Dependence for Hard Disks", *Journal of Chemical Physics* **46**, 686 (1967).